



Model Fertility Schedules: Variations in The Age Structure of Childbearing in Human Populations

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CURRENT ITEMS

MODEL FERTILITY SCHEDULES: VARIATIONS IN THE AGE STRUCTURE OF CHILDBEARING IN HUMAN POPULATIONS

A research project* recently undertaken at the Office of Population Research was an examination of the roots of a

basic integral equation in the theory of stable populations:

$$\int_a^{\beta} e^{-ra} p(a) m(a) da = 1.0$$

The aim was to examine the nature of the roots for a set of net fertility functions expressing the full variety of fertility experience to be found in large human populations. One segment of the project was an attempt to create a family of model fertility schedules encompassing the full range of human experience, an attempt that culminated in the tables presented here.

Several sets of model tables have been developed representing in different ways and in different detail of coverage typical age patterns of mortality found in human populations at every recorded level of mortality (United Nations 1955; Coale and Demeny 1966; Ledermann 1969; Brass 1971). A single model schedule of first marriage frequencies (with a corresponding model schedule of the proportion ever married by age, and also a schedule of person years lived in the ever married state) has been found to fit a wide range of age patterns of nuptiality, given the proper choice of parameters specifying the origin and the appropriate horizontal and vertical scales for the standard nuptiality function (Coale 1971).

In Appendix B are printed a set of model age-specific fertility schedules analogous to the earlier model tables of mortality and nuptiality. These schedules represent age *patterns* of fertility rather than the level of fertility. Since the sum of the tabulated fertility rates, taken over all reproductive ages, is 1.0, age-specific fertility rates can be calculated by multiplying each model rate by an actual population's total fertility rate. In these new tables the fertility in each single year of age is calculated as the product of a number representing the proportion cohabiting at that age and a number representing the age-specific fertility of those who cohabit. By such combinations we have been able to construct schedules that we believe express essentially the full range of age structures of fertility likely to be found in large human populations. The source of this belief is, first of all, the regularity, both in the age pattern of nuptiality, and in the variation of marital fertility with age, noted in an earlier article (Coale 1971). The further and sounder basis for the belief in the validity of the model fertility schedules is their extraordinarily close fit to various accurately recorded fertility schedules of rad-

*This project was conducted as part of a graduate course in mathematical demography at Princeton taught by Donald McNeil, Ansley Coale, and Jane Menken, during the fall semester of 1973-1974. It was a joint research project involving students and faculty, undertaken in lieu of individual research papers.

ically different form in terms of mean age, standard deviation, and symmetry or asymmetry. The fit is described and graphically illustrated at a later point.

The text that precedes the tables includes: 1) a description in general terms of the basis of the model schedules of fertility; 2) the presentation of relevant details of the two constituent functions that are multiplied together to form the schedules; 3) a discussion of the fit of the schedules, including their suitability given the existence of such empirical factors as extramarital fertility, dissolution of marriage, and rapid changes in nuptiality; 4) a brief discussion of the advantages of model schedules based on the combination of two functions; and 5) an indication of some of the applications of the schedules, including instructions for locating, by interpolation, the most appropriate set of age-specific rates.

The Basis for the Model Schedules of Fertility

The basic assumption upon which the model schedules are calculated is that fertility conforms to the structure by age created by multiplying together two model subschedules: a sequence of model proportions ever married at each age and a model schedule of marital fertility. Thus, if the proportion ever married at age a in the model schedule of nuptiality is $G(a)$, and the proportion of married women at age a experiencing a live birth in the model schedule of marital fertility is $r(a)$, age-specific fertility is $f(a) = G(a) \cdot r(a)$. This construction applies exactly to a hypothetical population in which there is no fertility outside marriage, and no dissolution of marriage before the end of the childbearing span of ages. But it also duplicates quite adequately the age structure of fertility in actual populations through the selection of a $G(a)$ that differs slightly from the proportion ever married in the actual population, and of an $r(a)$ that differs slightly from the actual marital fertility schedules.

The representation $f(a) = G(a) \cdot r(a)$ makes possible the calculation of model fertility schedules from three specified parameters—two parameters required to specify a model schedule of proportions ever married, and one parameter required to specify a model schedule of marital fertility.

Age Structure of the Proportion Ever Married, $G(a)$, Specified by Two Parameters

First-marriage frequencies, defined as the number of first marriages in a short age interval divided by the number of persons in that interval, have been shown to conform to a curve of the same shape in different populations (or more precisely in different cohorts). What differs from population to population is the age at which first marriage begins, the duration of the age span within which the majority of the marriages occur, and the proportion of the survivors in the cohort who, at advanced ages, have been married at some time. The similarity in structure of the age distribution of first marriages in different cohorts is analogous to the common shape characterizing different normal (Gaussian) distributions, which are alike only when the mean (location), standard deviation (horizontal scale), and vertical scale (number of cases, or size of population) are specified.

If the effect of differential mortality by marital status on the proportion ever married is neglected, the existence of a standard distribution of first marriage frequencies implies a standard curve describing the proportion ever married in different cohorts. The *form* of the curve is standard, but there are differences, of course, in the starting age of a tangible proportion ever married, in the pace at

which the curve rises and in the ultimate proportion experiencing marriage—the proportion ever married by the age at which first marriage rates have fallen essentially to zero. If the standard proportion ever married x years after first marriages begin is $G_s(x)$, in any cohort $G(a) = C \cdot G_s((a - a_0)/k)$, where C is a factor determined by the ultimate proportion ever married, a_0 is the age at which first marriages begin, and k is the scale factor expressing the number of years of nuptiality in the given population equivalent to one year in the standard population. If k is 1.0, first marriages occur at the same pace as in the nineteenth-century Swedish population that served as the basis of the standard; if k is 0.5, or one-half, first marriages occur at twice the pace of the standard. Specifically, according to the standard schedule half of the population that will ever marry has experienced first marriage ten years after the earliest age at which a consequential number of first marriages occur; if k is equal to 0.5, one-half the cohort has experienced first marriage five years after a_0 .

The standard proportions ever married were published in an earlier article (Coale 1971), but for computational convenience, we have calculated $G(a)$ from a closed-form analytical expression for first marriage frequencies developed by Donald R. McNeil (Coale and McNeil 1972). This expression is:

$$(1) \quad g(a) = (0.19465/k) \exp \{(-0.174/k)(a - a_0 - 6.06k) - \exp [(-0.2881/k)(a - a_0 - 6.06k)]\}$$

No analytical expression for $G(a)$ has been found, but $G(a)$ can be calculated by numerical integration of $g(a)$, since $G(a) = \int_{a_0}^a g(x)dx$. This representation of $G(a)$, with appropriate estimates of a_0 and k , provides an approximation of the proportion ever married in a cohort, if multiplied by a scale factor to allow for the particular proportion ultimately experiencing marriage. However, since the standard schedules of fertility that we have constructed represent only the age pattern of fertility and not the level, the proportion ultimately marrying is omitted here. Only the age of initiation and the pace of first marriages affect the structure of fertility; the proportion remaining celibate influences the level but not the age pattern of fertility.

The Age Structure of Marital Fertility, $r(a)$, Specified By a Single Parameter.

Louis Henry found that there is a characteristic pattern of marital fertility in populations in which there is little or no voluntary control of births. He defined voluntary control as behavior affecting fertility that is modified as parity increases, and the absence of control—natural fertility—as behavior, whether affecting fertility or not, that is the same no matter how many children have been born (Henry 1961). The regularity in marital fertility that makes possible a single-parameter set of schedules is this: marital fertility either follows natural fertility (if deliberate birth control is not practiced), or departs from natural fertility in a way that increases with age according to a typical pattern. In a population in which fertility is voluntarily controlled, the ratio of marital fertility at each age, $r(a)$, to a schedule of natural fertility, $n(a)$, is given by:

$$(2) \quad r(a)/n(a) = M \exp (m \cdot v(a))$$

The factor M is a scale factor expressing the ratio $r(a)/n(a)$ at some arbitrarily chosen age. Since we are concerned only with the age pattern of fertility (not its level), the value of M (like the value of the factor C in the model schedule of proportion ever married) is of no significance for the construction of our fertility schedules. The function $v(a)$ expresses the tendency for older women in populations practicing contraception or abortion to effect particularly large reductions of fertility below the natural level.

Model schedules of $r(a)$ are required at single years of age over the full range at which there is found both 1) a non-zero proportion cohabiting, and 2) non-zero marital fertility. The two functions $n(a)$ and $v(a)$, assumed to be invariant, must therefore be estimated by single years of age; the requisite family of model schedules is then obtained by assigning values to m , from zero, in which case $r(a)$ equals $n(a)$, to a maximum expressing the greatest likely departure of fertility from the age pattern of natural fertility resulting from a very high degree of voluntary control of births.

The functions $n(a)$ and $v(a)$ were derived from empirical data. There were two steps in the derivation: first, the estimation of approximate values of $n(a)$ and $v(a)$ by five-year age intervals above age 20, and second, determination of single-year values by freehand interpolation above age 20 plus extension to ages below 20 on somewhat arbitrary common sense principles.

Seven values of $n(a)$ at ages 20-24 through 45-49 were derived by calculating the arithmetical average of schedules designated by Henry as natural (Henry 1961). Henry's schedules begin at 20 because premarital conceptions have a large and irregular effect on teenage marital fertility. Ten schedules of natural fertility were averaged after discarding schedules known to be based on surveys in which age misreporting was especially prevalent and might have distorted the pattern of fertility. The effect of this selection (compared to the acceptance of all schedules listed by Henry) is minor, since the age pattern of all of those listed is broadly similar.

Seven values of $v(a)$, at ages 20-24 through 45-49, were obtained by calculations employing the marital fertility schedules listed in the United Nations Demographic Yearbook for 1965 (United Nations 1966). Again, schedules known or suspected to be distorted by age misreporting or other forms of faulty data were discarded. Each of the forty-three schedules not eliminated on this basis were provisionally accepted as embodying, each in its own degree, the typical pattern of departure from natural fertility.

For the i^{th} schedule an individual $v_i(a)$ can be calculated by setting $m = 1.0$ in equation (2). For the i^{th} schedule we find

(3)
$$v_i(a) = \log [r_i(a)/(M \cdot n(a))]$$

M is chosen so that $v_i(a)$ is zero for the age interval 20-24. The arithmetical average of the forty-three values of $v_i(a)$ in each of the seven age intervals was then defined as $v(a)$ for each interval. The values of $n(a)$ and $v(a)$ are as follows:

	20-24	25-29	30-34	35-39	40-44	45-49
$n(a)$	0.460	0.431	0.396	0.321	0.167	0.024
$v(a)$	0.000	-0.316	-0.814	-1.048	-1.424	-1.667

The function $v(a)$ calculated in this way can be validated by substituting the tabulated values in equation (2) and seeing how well the result fits each marital fertility schedule. A value of M is chosen that equates $M \cdot n(a)$ with $r(a)$ at ages 20-24. One way of getting a visual impression of how well $v(a)$ fits a given marital fertility schedule is to calculate a separate value of m for each age interval. If equation (1) were fully valid, and $v(a)$ appropriately estimated, the separately determined values of m for age intervals 25-29 through 45-49 would all be the same. The sequence of m 's calculated for the forty-three empirical marital fertility schedules is not in every instance highly uniform. However, the set of m 's for most marital fertility schedules falls on a reasonably level plateau, and the difference in level of m between different populations is quite evident (see Figure 1).

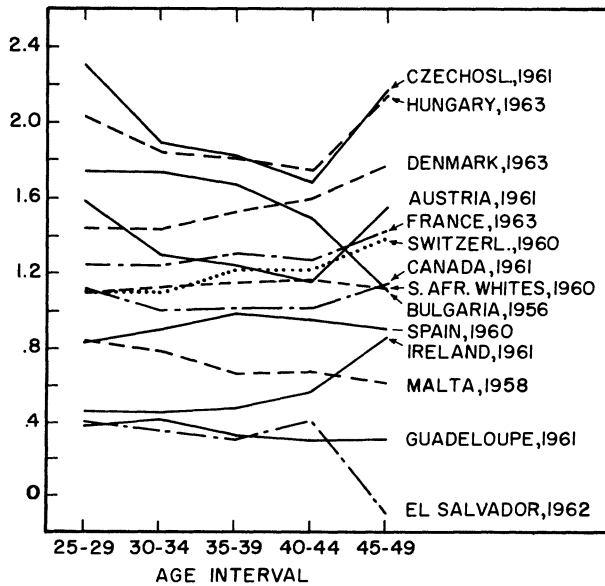


Fig. 1. Values of m , where $m = \log[r(a)/(M \cdot n(a))]/v(a)$, for selected marital fertility schedules

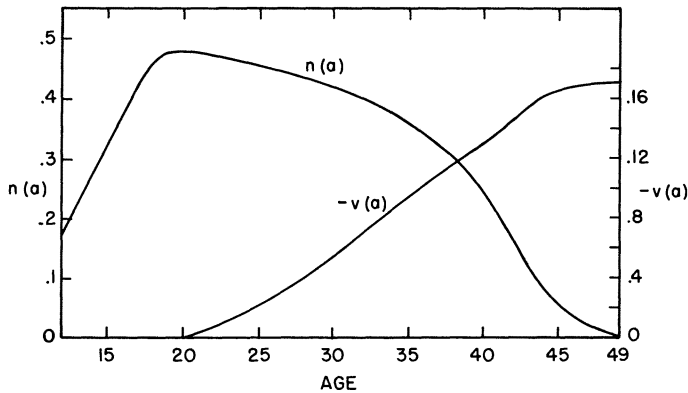


Fig. 2. Values of $n(a)$ (natural fertility), and $v(a)$ (logarithmic departure from $n(a)$)

Single-year values of $n(a)$ and $v(a)$ are shown in Figure 2, and tabulated as part of the FORTRAN program in Appendix A. The hand-fitted values of $n(a)$ above age 20 approximately match, in average value for each five-year interval, the values at five-year intervals listed earlier. The extension of $n(a)$ back to age 12 is based on general biomedical information that full reproductive capacity is reached a few years after menarche, and that the mean age at menarche varies from about 12 to 16 years in different populations. The particular choice of rates to represent $n(a)$ below age 20 is not of major importance because of the dominant role of $G(a)$ in determining the rise of age-specific fertility with age.

Values of $v(a)$ at single ages were chosen so that their sum over five-year age intervals matched (above age 25) the values at five-year intervals given earlier. To avoid a sharp change in the neighborhood of age 25, non-zero values were assumed to begin at age 20.

With single-year values of our three functions, we have the means of calculating a full range of fertility schedules for hypothetical populations in which there is no illegitimacy and no marital dissolution, and in which marriage begins at various initial ages and occurs over various age spans, and in which marital fertility ranges from the gradual decline with age characteristic of natural fertility to the much steeper decline characteristic of populations in which there is extensive control of fertility within marriage. The age pattern is given by equation (4):

$$(4) \quad f(a) = G(a)n(a)e^{m \cdot v(a)}$$

where $f(a)$ is age-specific fertility, $G(a)$ is the proportion ever married (in a population where first marriage occurs according to a schedule characterized by selected values of the parameters a_0 and k), $n(a)$ is natural fertility, $v(a)$ is the characteristic pattern of departure from natural fertility, and m is the extent of that departure.

Model Schedules of Age-Specific Fertility, and Their Similarity to the Age Pattern of Fertility in Actual Populations

In actual populations, of course, births occur outside of marriage as well as within, and the proportion of the population currently married differs from the proportion ever married because of the presence of the widowed and divorced. However, the structure of fertility in an actual population may closely resemble that in a hypothetical population with no marital dissolution or extramarital fertility if the latter population has slightly different parameters of nuptiality and marital fertility from those found in the actual population. The effect of illegitimate births and of premarital conceptions on the age structure of fertility is equivalent to a schedule of first marriages that is slightly different from the observed one at early ages; the effect of illegitimate births at the older ages is equivalent to a slight increase in marital fertility at those ages. The proportion of the ever married population that is widowed and divorced rises monotonically with age, thus reducing fertility toward the end of childbearing in a way that is topographically similar to the effect of $v(a)$ on marital fertility. In other words, it is probable that the standard schedule of first-marriage frequencies, with a suitable choice of initial age and pace of occurrence of first marriages, can serve as a usable surrogate for the age of entry into sexual union (including unions that do not in

fact involve marriage), and that modification of natural fertility by the proper choice of m by which to multiply $\nu(a)$ can serve to approximate the effect both of marital dissolution in reducing the fraction married at higher ages and of control of fertility on marital fertility. On the provisional assumption that such is the case, we have calculated a large array of model fertility schedules by single years of age; each schedule is composed of the product of an estimated proportion ever married and of marital fertility in each single-year age interval. The starting age of nuptiality was allowed to range from 12.5 to 18 years; the pace of marriage from 56 percent of the pace ($k = 1.7$) to five times the pace ($k = 0.2$) in the Swedish standard nuptiality schedule. The value of m was permitted to range from zero (natural fertility) to 3.9, on a scale in which 1.0 is the average value for forty-three schedules in the 1965 Demographic Yearbook. A total of 795 model schedules was tabulated. Each schedule has been normalized so that the sum of the fertility rates at all ages is 1.0; the schedules embody only an *age pattern* of fertility and carry no implication with respect to total fertility.

The tabulated schedules have been selected to produce mean ages at integral values from 24 to 34 years and values of standard deviation (achievable within the stipulated limits of the three underlying parameters) at intervals of half a year. The range of standard deviation is from 4.0 to 7.5, but some combinations (e.g. standard deviations of 7.0 or 7.5 with a mean age of 25) could not be attained within the limits of the three controlling parameters.

When a_0 was 15.0 or more, the single-year rates under age 20 were modified to conform to an observed feature of reliably recorded single-year schedules; non-zero fertility rates typically begin at about age 15 even when marriage begins relatively late. Positive fertility rates at ages 15 and 16 in such populations are probably the result primarily of extramarital conceptions that occur to a small number of adolescents. The requisite modification was achieved as follows: the value of fertility at exact age 20 and the cumulated value of fertility up to age 20 were accepted as initially calculated from equation (3). Values of n and R were found such that $f(a)$ equals $R(a - 15)^n$ matches the calculated value at age 20, and such that $R \int_{15}^{20} (a - 15)^n da$ matches cumulated fertility (as calculated) up to age 20.

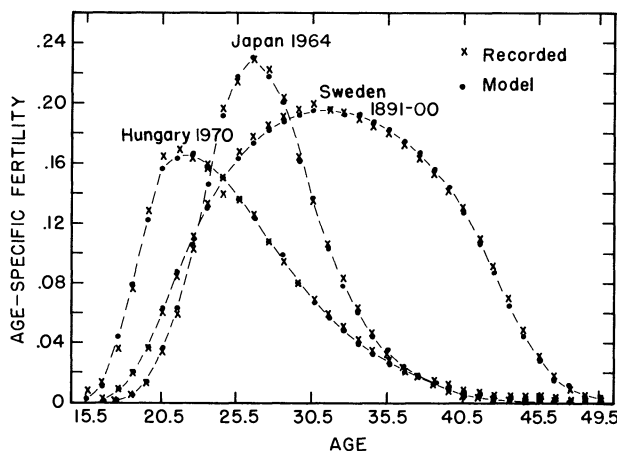


Fig. 3. Age-specific fertility rates of three populations fitted by model fertility schedules

A crucial question is whether this family of model fertility schedules provides a close fit to the fertility of actual populations. We have tried to determine how well the model schedules operate by finding a schedule (through interpolation among the printed values) that matches each of a number of recorded schedules in terms of the mean age and the standard deviation and the ratio of the average value of fertility in the interval from ages 15 to 20 to the average value from ages 20 to 25. Figure 3 shows the goodness of fit for three selected fertility schedules recorded by single years of age.

The schedules were chosen because they had the lowest and highest mean ages (Hungary, 1970, and Sweden, 1891-1900), and the lowest standard deviation (Japan, 1964) among the single-year fertility schedules that we examined; in spite of the fact that the schedules fitted are extreme, the fit in every case is quite close. In fact the absolute value of the area between the model schedule and the recorded rates is in each instance less than 2.5 percent of the total area under either curve. We have fitted a number of other recorded fertility schedules with equal success.

Figure 4 shows the structure of fertility that results when entry into cohabitation is early and rapid or late and gradual, combined with natural fertility, and with fertility that is highly controlled. In interpreting Figure 4 the reader must keep in mind the normalization of each schedule so as to produce an arbitrary total fertility of 1.0. The figure illustrates the distribution of fertility by age, not differences in level of fertility associated with age patterns. Actually, a schedule incorporating natural fertility would be expected to have at least as high fertility at every age as a schedule with the same a_0 and k and positive values of m . In Figure 5 two schedules with the same nuptiality but different values of m are shown, when the final proportion married is set at 1.0, and natural fertility is given a maximum value of 0.477.

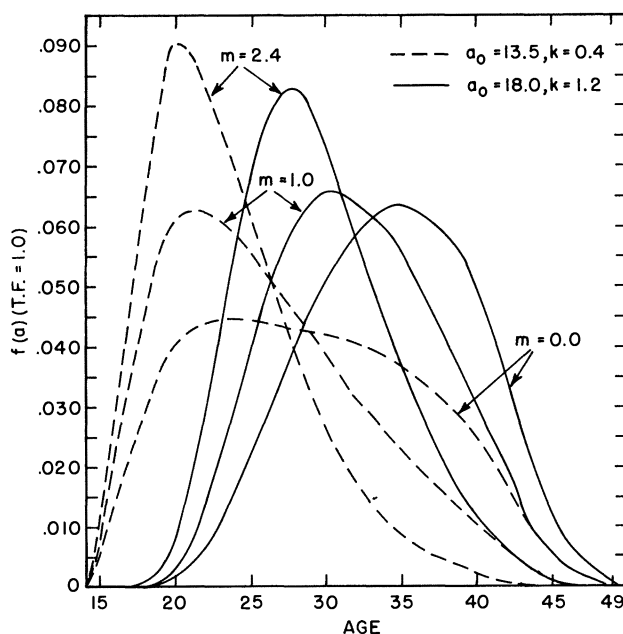


Fig. 4. Model fertility schedules, total fertility = 1.0. Combinations of early marriage with various degrees of fertility control and late marriage with various degrees of control

Suitability of the Model Fertility Schedules when Nuptiality is Changing

One of the two basic components of the model fertility schedules—the standard schedule of first-marriage frequencies—logically fits the experience of a cohort as it moves through life; it cannot match the proportion ever married by age in a cross section during a period of rapid changes in nuptiality. In fact, during such a

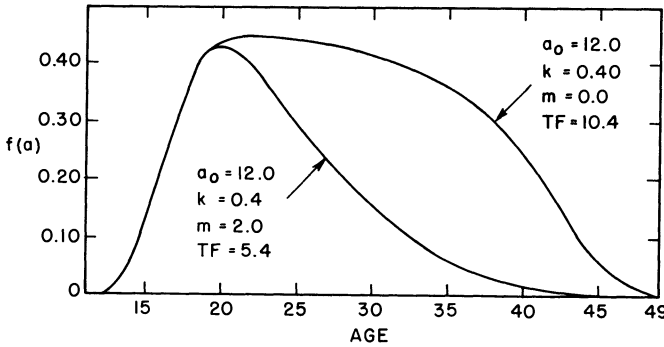


Fig. 5. Age-specific fertility schedules, proportion ultimately ever married 1.0, marital fertility given by $n(a) \cdot \exp(m \cdot v(a))$

period, there may be such peculiarities as a proportion ever married at age 30 higher than at 40. But an examination of long sequences of Swedish and Danish period and cohort marital fertility schedules reveals that the second basic component—a set of model schedules of marital fertility—fits cross-sectional experience better than it fits cohort experience. Thus one of the components is appropriate to the construction of model schedules for periods, and the other component is not. In particular, we can expect difficulties in fitting the model schedules to actual experience when nuptiality is changing rapidly.

The good performance of a model schedule in matching fertility for Japan, 1964 (Figure 3), shows that this logical defect does not necessarily impair the capacity of the model schedules to duplicate real age patterns. However, the fit is achieved with a fertility schedule embodying an implied mean age of first marriages (32.4 years) that bears no relation to the actual mean age at marriage in Japan (about 24 years). In contrast, the model schedule fitted to Hungary, 1970, implies a mean age at first marriage within 0.4 years of the recorded mean. Thus the model schedules fit quite well even when the assumptions they incorporate are violated; however, the parameters (a_0 , k , and m) that in periods of constant nuptiality approximately specify the age pattern of entry into cohabitation and the departure of marital fertility from the “natural” pattern cannot be so interpreted in a period of rapid change.

Advantages of the Model Schedules

A virtue of this set of model schedules as compared to fitted schedules that are based on conventional frequency distributions such as the log normal, or one of the Pearson curves (Tekse 1967; Talwar 1970; Mitra and Romaniuk 1972; Romaniuk 1973; Talwar 1974), is that the model schedules incorporate combinations of intuitively understandable demographic factors. The validity of this basis for

constructing model tables is confirmed by the goodness of fit to a variety of accurately recorded schedules. The model tables have the further advantage of describing in detail age patterns of fertility that are widely experienced but seldom recorded. Early marriage has been combined with natural fertility in many populations, but this combination has usually occurred in the absence of accurate registration of birth by age of mother; consequently few instances of this age pattern of fertility have been observed in detail. The model tables provide a useful tool of estimation for such populations.

Some Possible Uses of Model Fertility Schedules

It is hoped that these model fertility schedules will prove useful in a number of practical, analytical, and heuristic ways, only some of which can be foreseen at this early stage. One practical purpose is to provide estimated single-year fertility rates for populations in which age-specific fertility is tabulated only by five-year age intervals. The tables have been arranged to make it possible to locate a model fertility schedule on the basis of a known mean and standard deviation plus the ratio of fertility at ages 15-19 to fertility at ages 20-24 (labeled R_1 in the model tables). (In calculating the standard deviation from data given by five-year age intervals it is necessary to allow for Shepherd's correction or to subtract 2.083 from the calculated variance.)

Fitting a Model Schedule to an Observed Schedule: England and Wales, 1965

It is usually possible to calculate a model fertility schedule matching observed values of mean age, standard deviation, and R_1 by employing the weighted average of no more than three tabulated schedules. Suppose the given values of mean age and standard deviation lie between $\hat{\bar{x}}$ and $\hat{\bar{x}} + 1.0$, and $\hat{\sigma}$ and $\hat{\sigma} + 0.5$, respectively, as shown in Figure 6. Since model tables are tabulated for integral values of the mean age, and for standard deviations at intervals of 0.5 years, tabulated fertility schedules generally exist at combinations of \bar{x} and σ^2 found at all four corners of the rectangle in Figure 6. These schedules are examined to see if they include

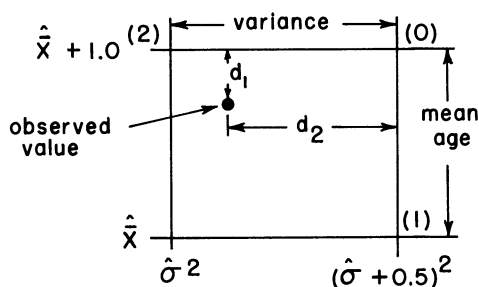


Fig. 6. Guide to interpolation to determine weights for calculating model fertility schedule as weighted average of tabulated schedules

schedules with values of R_1 close to the observed R_1 . Usually at least three of the sets of tabulated schedules at the four corners provide such values of R_1 . Let us designate the three positions at which correct R_1 's occur as positions 0, 1, and 2, employing 0 for the position sharing the standard deviation of position 1 and the mean age of position 2. Let d_1 be the distance from the observed mean to the mean at position 0, and d_2 the distance of the observed variance from the variance at

position 0, as a fraction of the distance 0 to 2. (Cf. Figure 6.) Then if weights $W_1 = d_1$, $W_2 = d_2$, and $W_0 = 1.0 - d_1 - d_2$ are applied to schedules at positions 1, 2, and 0, the resultant weighted average of fertility rates constitutes a schedule that has the observed mean and variance.¹

To match the observed value of R_1 to a very close approximation, it is usually sufficient to choose judiciously from the schedules available at positions 0, 1, and 2, choosing two schedules with R_1 's on one side of the observed R_1 , and one schedule with an R_1 on the other side, paying due attention to the weights W_0 , W_1 , and W_2 . The aim, of course, is to select the schedules so that the weighted average of the R_1 's matches the observed R_1 .² As an example, consider the fertility schedule for England and Wales 1965, with $\bar{x} = 27.269$, $\sigma = 5.672$, and $R_1 = 0.248$. Position 0 is $\bar{x} = 27.0$, $\sigma = 6.0$, position 1 is $\bar{x} = 28.0$, $\sigma = 6.0$, and position 2 is $\bar{x} = 27.0$, $\sigma = 5.5$. The value of d_1 is $0.269/1.0$; the value of d_2 is $[(6.0)^2 - (5.672)^2] / [(6.0)^2 - (5.5)^2] = 0.6658$. The adjustment³ to d_2 is $(2.69) \cdot (731) / 5.75 = 0.342$. Thus $W_1 = 0.269$, $W_2 = 0.700$, and $W_0 = 0.031$. At position 0 ($\bar{x} = 27.0$, $\sigma = 6.0$) we choose $R_1 = 0.2424$ (too small); at position 1 ($\bar{x} = 28.0$, $\sigma = 6.0$), $R_1 = 0.2494$ (too large); at position 2 ($\bar{x} = 27.0$, $\sigma = 5.5$), $R_1 = 0.2478$ (too small). The weighted average is $(0.031) (0.2424) + (0.269) (0.2494) + (0.700) (0.2478) = 0.2480$. The cover chart shows the resultant fit to the recorded schedule.

Fitting a Model Schedule to Observed Average Parities in a Developing Country: Peru, 1960

Another practical use is to locate a model fertility schedule for a population of a less developed country for which the only information is a sequence of reported average parities by five-year age intervals. Suppose it may be assumed that fertility has been approximately constant in recent years, and that fertility is either natural fertility or subject to only a slight degree of control. It is common knowledge that reported parity falls off with age beyond a certain point and is generally understated for older women. A plausible conjecture about reporting of parity in populations in which the responses are deficient is that younger women give a fairly full and accurate report of the number of children ever born to them, and that older women fail to report all of the births that have occurred to them mainly because of a failure to understand that they should include children who have grown up and left home. In other words the parity reported by women up to about age 30 can be considered relatively accurate.

With the help of Figure 7, it is possible to determine the values of a_0 and k that would yield specified combinations of the ratios $PAR\ 1$ (average parity 15-19)/(average parity 20-24) and $PAR\ 2$ (average parity 20-24)/(average parity 25-29), with $m = 0.0$ (natural fertility), $m = 0.2$ (very moderate control of fertility) and $m = 0.4$ (quite moderate control of fertility). The FORTRAN program in Appendix A can then be used to calculate a model fertility schedule with an age structure that 1) matches the observed sequence of average parities up to age 30, and 2) incorporates either no departure or only a slight departure from natural fertility at the higher ages. The schedule is printed out at single years of age; average parity at ages 15-19, 20-24, and 25-29 is also provided. The model schedule yields a total fertility of 1.0; hence the ratio of average parity at ages 25-29 (or at either of the other tabulated age intervals) recorded for the popula-

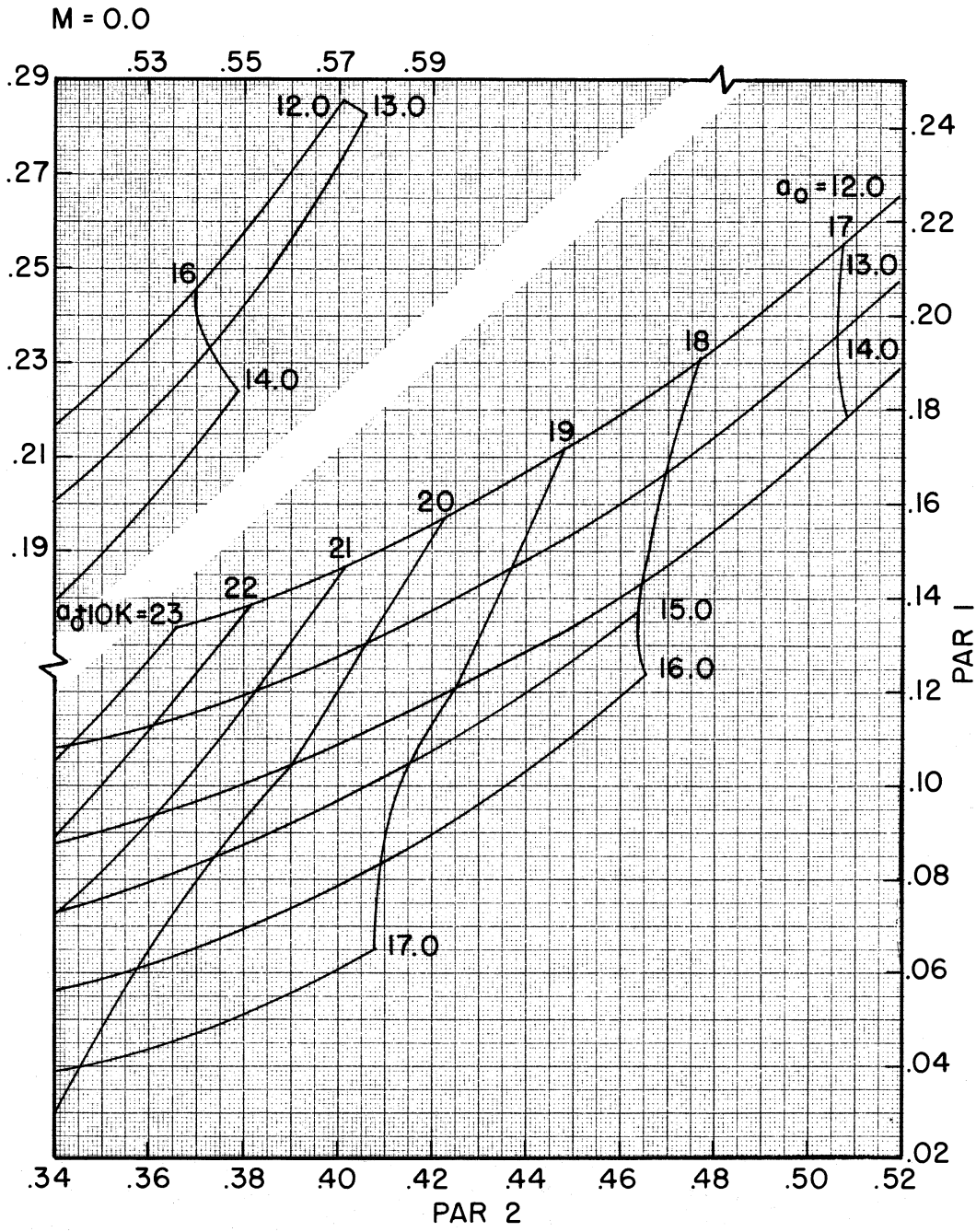


Fig. 7. Locus of combinations of *PAR 1* (average parity 15-19/average parity 20-24) and *PAR 2* (average parity 20-24/average parity 25-29) giving specified values of a_0 and $a_0 + 10k$, for $m = 0.0, 0.2$, and 0.4 .

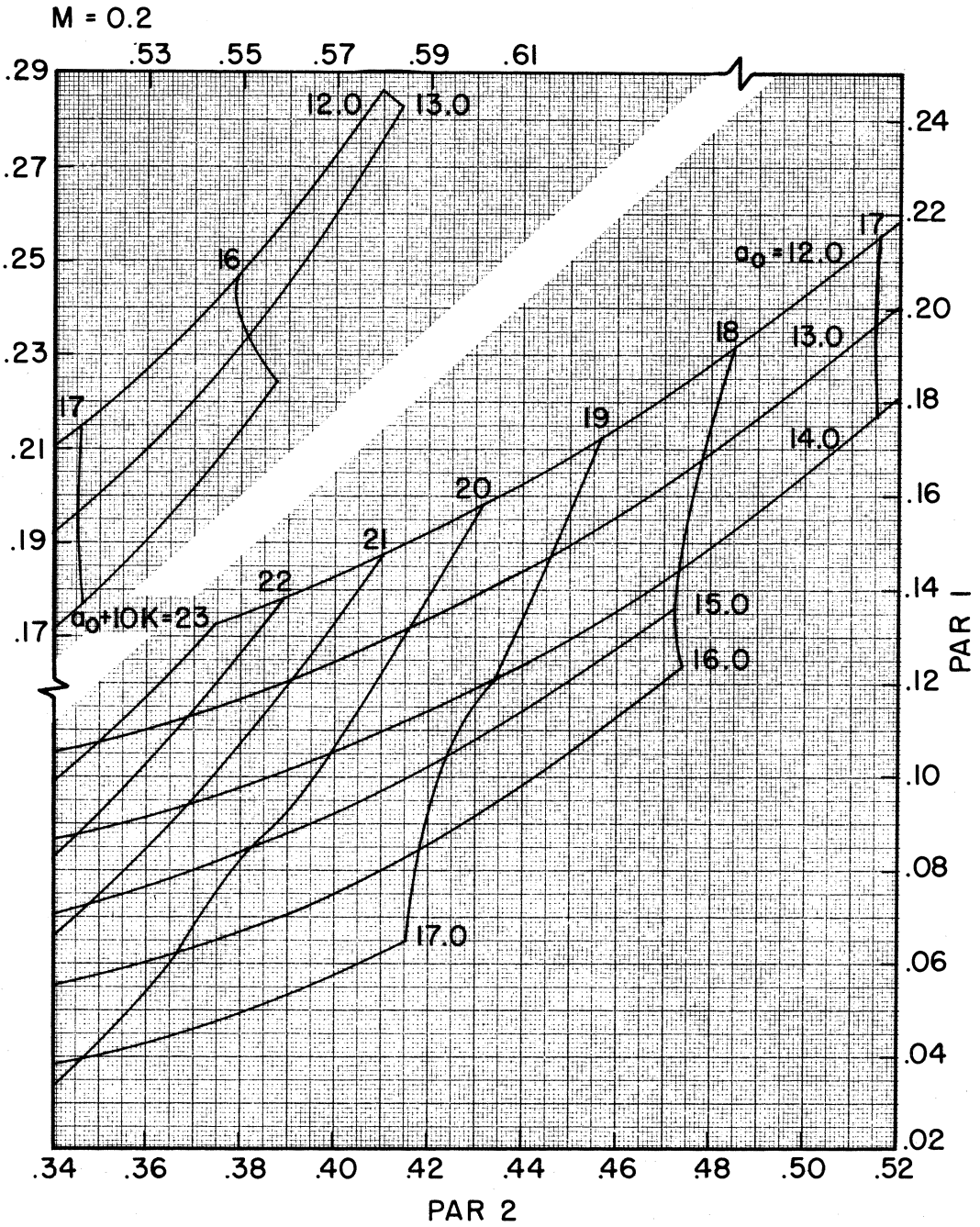


Fig. 7 (cont.)

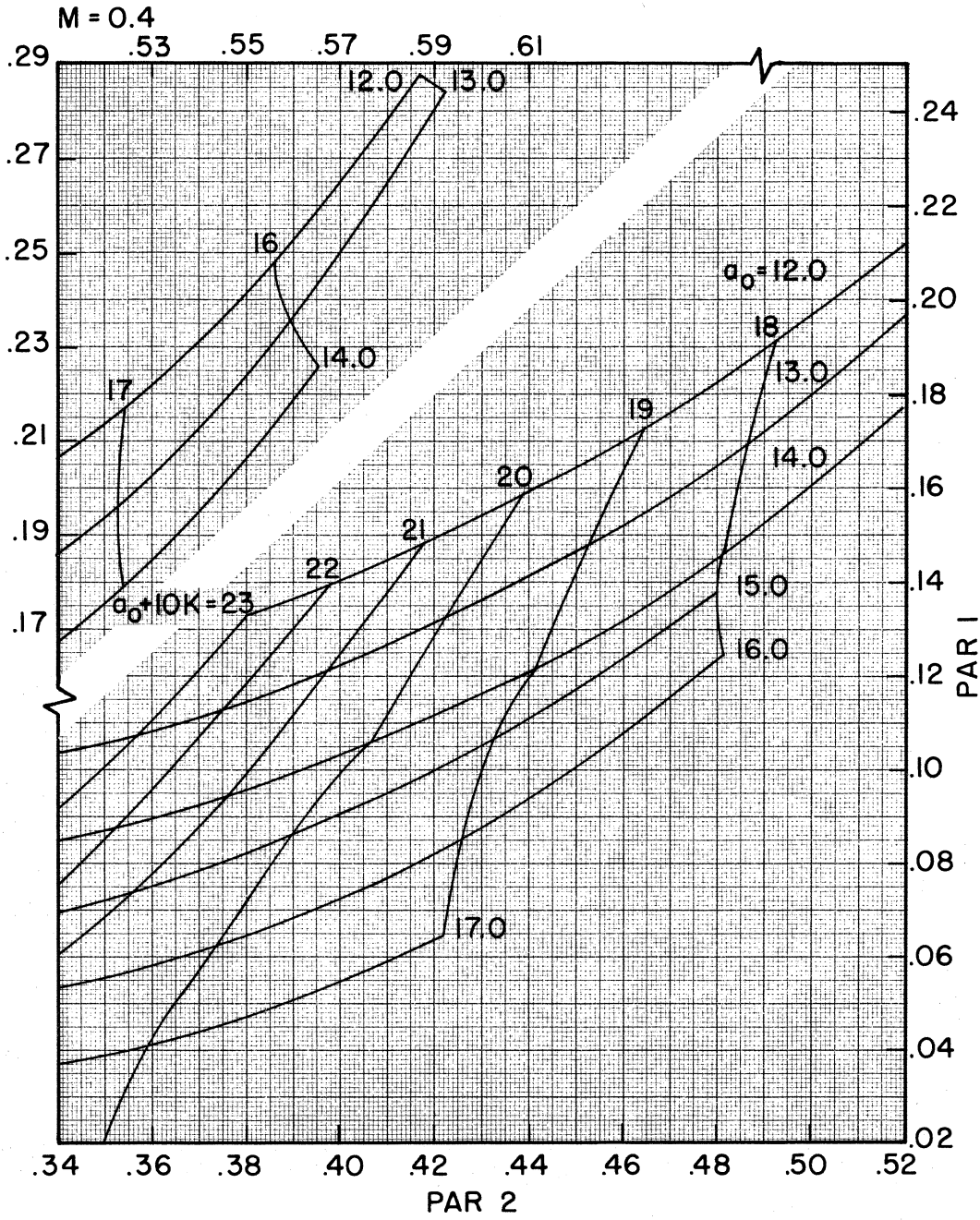


Fig. 7 (cont.)

tion to average parity at that age in the model schedule provides an estimate of total fertility of the population. This ratio is also the multiplier required to convert the model age-specific fertility rates to the level prevailing in the population.

In Figure 7 it is possible to estimate, by visual interpolation, values of a_0 and $a_0 + 10k$ corresponding to specified combinations of *PAR* 1 and *PAR* 2, for $m = 0.0$, $m = 0.2$, and $m = 0.4$. The figure displays $a_0 + 10k$ rather than simply k as the second variable because the loci of constant $a_0 + 10k$ are more nearly orthogonal to the loci of a_0 than are the loci of k itself; this is not surprising, since $a_0 + 10k$ is the median age of first marriage in a first marriage distribution specified by the parameters a_0 , k . To find the values of a_0 and k consistent with given values of *PAR* 1 and *PAR* 2, locate the given *PAR* 1 and *PAR* 2 in one of the panels of Figure 7, and estimate the fractional distance of this position between two values of a_0 , and two values of $a_0 + k$. For example, *PAR* 1 and *PAR* 2 are 0.1424 and 0.4514 for Peru, 1960. When $m = 0.2$, this point lies at about $a_0 = 13.4$, $a_0 + 10k = 18.7$. When $a_0 = 13.4$ and $a_0 + 10k = 18.7$, $k = 0.53$. Hence one combination of parameters that produces a schedule with Peru's *PAR* 1 and *PAR* 2 is a schedule with $a_0 = 13.4$, $k = 0.53$, and $m = 0.2$. Other values of a_0 and k (13.2 and 0.58) would serve if $m = 0.4$ or (13.66) and 0.48) if $m = 0.0$.

These three model schedules, adjusted to yield the average parity at ages 25-29 recorded for Peru, are shown in Figure 8a. The estimates of total fertility implied by the three are 5.94, 6.30, and 6.72. Supplementary information for Peru makes it possible to select one of these schedules as optimal: the mean age of the schedule calculated from Peru's incomplete register of births by age of mother is 29.50 years, closely matching the mean age of the model schedule with $m = 0.20$. Total fertility for this model schedule, adjusted to match recorded parity at 25-29, is 6.30; total fertility according to registered births is 5.09, indicating a completeness of registration of 80.8 percent. The age structure of the model schedule, chosen primarily on the basis of average parities recorded in the census, agrees well with the structure of fertility indicated by registered births (Figure 8b).

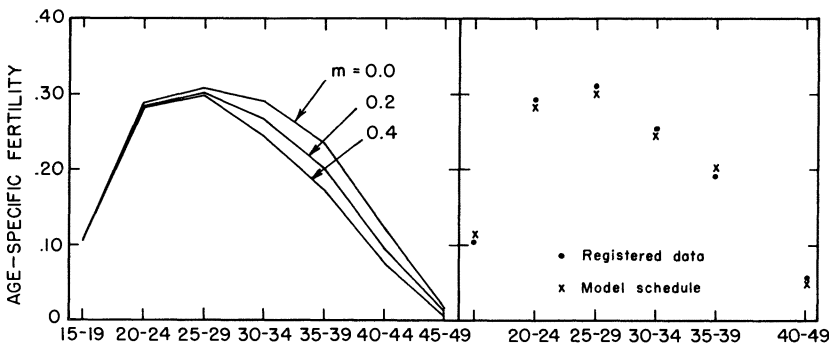


Fig. 8a. Model fertility schedules, by five-year age intervals, matching average parity in Peru, 1960, at ages 15-19, 20-24, and 25-29.

Fig. 8b. Model fertility schedule, $M = 0.20$, compared with registered rates for Peru (adjusted for underregistration)

A similar calculation for Mexico, 1960, produces a similarly close agreement; however, the best fit to the mean age based on registered births is provided by the model schedule with $m = 0.4$; this model schedule adjusted to the recorded average parity at 25-29 yields a total fertility of 6.12, 1 percent *less* than total fertility calculated from registered births. By this test, the registration of births in Mexico is seen to be complete.

In the absence of extensive registration the mean age of the fertility schedule is not available, and it is necessary to guess the appropriate value of m , on the basis of general knowledge. For example, on the basis of the above calculations, a value of 0.2 to 0.4 seems a sensible choice for a Latin American population in which no major decline of fertility has occurred.

Another category of uses of these model tables is analytical. It was really for analytical reasons that we embarked on their construction. The application of these schedules in an exploration of the nature of the complex roots

$$\int_{\alpha}^{\beta} e^{-ra} p(a) \cdot m(a) da = 1$$

will be reported elsewhere in a paper on that topic. In addition, the model fertility schedules by single years of age provide a firmer basis for calculation of adjustment factors to be used in the Brass-Sullivan approach to the estimation of infant and child mortality from data on the proportion dead among children ever born to women of different ages. In Brass's original version of these procedures, adjustment factors for converting proportions dead to ${}_nq_0$'s were derived by assuming a fertility function consisting of a polynomial of fixed structure that varied in its starting point. Sullivan determined the value of adjustment factors by constructing the adjustment required for each of a number of empirical fertility schedules by single years of age, and used regression analysis to determine the relationship between the needed adjustment factor and the parity ratios (*PAR 1* and *PAR 2*) discussed above. Sullivan was hampered by the scarcity of fertility schedules incorporating an early start of fertility, and attempted to remedy this deficiency by using fictitious fertility schedules incorporating a start one year earlier than that recorded in empirical schedules of fertility. The new model tables, which seem to fit empirical experience quite satisfactorily, provide a set of tables for the full range of likely human experience. It must be conceded that their use now in the calculation of Brass-type estimates of infant and child mortality would probably modify such estimates only slightly. However, the tables provide a more satisfactory basis for such calculations than the expedients employed earlier and it is hoped that in the future they will prove convenient for a variety of uses in analytical demography.

Description of Tables

The model fertility tables give age-specific fertility rates (per 1,000,000 women at each age), normalized so that the total fertility in each schedule is 1.0. The tables are arranged in ascending order of mean age, with ascending order of standard deviation with each mean age. For each value \bar{x} and σ , the tables are presented in ascending order of k . The ratio of average fertility at ages 15-19 to average fertility at ages 20-24, R_1 , generally is strictly monotonic increasing, but is

sometimes strictly monotonic decreasing, with increasing k . Also shown for each table are ratios of average parity at 15-19 to average parity at 20-24 (*PAR* 1) and average parity at 20-24 to average parity at 25-29 (*PAR* 2).

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NOTES

¹ The mean age of the weighted average of two schedules is the weighted average of the two means. The variance of the weighted average of two schedules *with the same mean* is the weighted average of the two variances. It is for this reason that the variance rather than the standard deviation is used for interpolation. However, the variance of the weighted average of two schedules with means that differ by 1.0, and the same variance, exceeds the common variance by $(W_1)(1 - W_1)$, where W_1 and $(1 - W_1)$ are the weights employed in interpolating for the mean age. (These statements can be verified by calculating the mean and variance of $Wf_1(x) + (1 - W)f_2(x)$.) Therefore, the variance of the interpolated schedule is slightly too large, and the weight W_2 should be modified by an increment $(W_1)(1 - W_1)/(\hat{\sigma}^2 + 0.25)$ since the difference between $\hat{\sigma}^2$ and $(\hat{\sigma}^2 + 0.5)^2$ is $\hat{\sigma}^2 + 0.25$. W_2 should be increased in this amount if position 2 is associated with the smaller variance; otherwise W_2 should be decreased by this increment. W_0 should also be readjusted so that the weights still add to 1.0.

² It is always possible to match the observed R_1 exactly by employing a weighted average of two schedules at one of the three positions, choosing the weights so that, for example, $W_0(R_1)_0 + W_1(R_1)_1 + W_2(R_1)_2 = R_1$, where $(R_1)_2$ is from a schedule at position 2 that is the weighted average of two schedules at that position. $(R_1)_2$ is chosen to equal $(R_1 - W_0(R_1)_0 - W_1(R_1)_1)/W_2$.

³ See Note 1.

APPENDIX A: Program for Computing a Model Fertility Schedule with Specified Values of a_0 , k , and m

This program in FORTRAN IV is self-contained, incorporating single-year values of $n(a)$ and $v(a)$, and including the calculations of $G(a)$ from a standard schedule of first marriage frequencies. The only data required are values of a_0 , k , and m (designated as AAA, AKK, and AMM in the only READ statement).

The values of $-v(a)$ and $n(a)$, natural fertility, begin at age 12.5 and extend to age 49.5 by simply listing 38 numbers.

No attempt was made to achieve elegance in programming. The program has the virtue that it has been debugged, and for all but expert programmers will save time.

```
C
C
DIMENSION EN(38), ZS(42), ZSS(500), H(38), V(38)
DIMENSION F(38), T(7), RR(5), CQ(9)

V(A), CENTERED ON THE MIDPOINT OF EACH YEAR OF AGE
DATA V/O., .0., .0., .0., .0., .0., .0., .004, .03, .06, .10, .15, .20, .25, .31,
1   .37, .44, .52, .60, .68, .76, .83, .90, .97, 1.04, 1.11, 1.18, 1.25,
1   1.32, 1.39, 1.46, 1.53, 1.59, 1.64, 1.67, 1.69, 1.70/

NATURAL FERTILITY, CENTERED CN THE MIDPOINT OF EACH YEAR OF AGE
DATA H/.175,.225,.275,.325,.375,.421,.460,.475,.477,.475,.470,
1    .465,.460,.455,.449,.442,.435,.428,.420,.410,.400,.389,
1    .375,.360,.343,.325,.305,.280,.247,.207,.167,.126,.087,
1    .055,.035,.021,.011,.003/
REAL*8 CQ/' PTY1',' PTY2',' PTY3',' PAR1',' PAR2','      12','      13',
1     '14','10-14'/
REAL*8 C(48)' NEAN','STDEV',' R1',' AO',' K',' M','      2'
1     '15','16','17','18','19','15-19','20','21','22','23','24','25','26','27','28',
1     '29','30-34','35','36','37','38','39','35-39','40','41','42','43',
1     '44','40-44','45','46','47','48',
1     '49','45-49'/
97 FORMAT(1H1,A5,5X,F8.4)
99 FORMAT(1X,A5,5X,F8.4)
888 FORMAT(3F10.4)
98 FORMAT(35(1X,A5,5X,F10.8/))
95 FORMAT(1X,A5,5X,F10.8)
DO 1234 L=1,38
1234 V(L)=-V(L)
DO 637 I=1,80
637 ZSS(I)=0.0
999 CONTINUE
READ(5,888,END=3) AAA ,AKK ,AMM

THE SECTION THROUGH STATEMENT 5 ESTABLISHES THE CUMULATIVE OF
THE CUMULATIVE EVER HARRIED SCHEDULE BY 0.1 YEAP INTERVALS WITH
ZERO ORIGIN
DO 73 J=1,420
73 ZS(J)=0.0
X=0.0
ZL=G(X,AKK)
DO 5 I=1,420
AI=I
X=AI/10.0
ZU=G(X,AKK)
ZS(I)=0.1*(ZU+ZL)/2.0)
ZL=ZU
IF (I.GT.1) ZS(I)=ZS(I)+ZS(I-1)
5 CONTINUE

THE SECTION THROUGH STATEMENT 4 TRANSFERS THE ORIGIN TO A0
DO 9 I=81,500
9 ZSS(I)=0.0
J=10.0*AAA
LAST=500-J
DO 4 I=1,LAST
J=J+1
ZSS(J)=ZS(I)
4 CONTINUE

BY AVERAGING THE CUMULATIVE OF THE CUMULATIVE EVER HARRIED
SCHEDULE FOR THE 100 VALUES IN EACH SINGLE YEAR OF AGE, THE
AVERAGE EVER HARRIED FOR EACH YEAR OF AGE IS ESTABLISHED
DO 25 I2=1,38
II2=120+10*I2
```

```

W=0.0
DO 24 K=1,10
24 W=W+0.5*(ZSS(II2-K+1)+ZSS(II2-K))
25 EM2(I2)=W/10.0
50 DO 35 I2=1,38
35 F(I2)=EM2(I2)*H(I2)*EXP(AMM *V(I2))

C      THE 15-19 SECTION OF THE AGE SPECIFIC FERTILITY SCHEDULE
C      ESTABLISHED IN STATEMENT 35 IS NOW TRANSFORMED BY FITTING
C      AN EXPONENTIAL HAVING CONTACT WITH THE AGE AXIS AT AGE 15 AND
C      ORDINATE AT AGE 20 AND AREA UNDER THE CURVE FROM 15-19 EQUAL
C      TO THAT OF THE ORIGINAL 15-19 SECTION. THIS TRANSFORMATION IS
C      NOT PERFORMED UNLESS A0 IS GREATER THAN 15

DO 1 IL=1,7
BB=0.0
DO 2 JL=1,5
KL=JL+5*(IL-1)+3
2 BB=BB+F(KL)
1 T(IL)=BB/5.0
FIRST=(F(1)+F(2)+F(3))/5.0
IP(AAA .LT. 15.0) GO TO 289
TT=T(1)*5.0
FR=.476*ZSS(200)
SS=FR*5.0/TT-1.0
CONS=FR/(5.0*SS)
A=1.0
DO 44 ML=1,5
RR(ML)=A**((SS+1.0)/(SS+1.0))*CONS
44 A=A+1.
F(4)=RR(1)
DO 46 M=2,5
L=M+3
46 F(L)=RR(M)-RR(M-1)
289 CONTINUE

C      THE SECTION THROUGH STATEMENT 37 ESTABLISHES THE MEAN, VARIANCE,
C      THE 3 PARITIES, AND R1

SUMF=0.
DO 222 I2=1,38
222 SUMF=F(I2)+SUMF
DO 333 J=1,7
333 T(J)=T(J)/SUMF
FIRST=FIRST/SUMF
SUM=0.
SUMSQ=0.
A=12.5
DO 33 I2=1,38
F(I2)=F(I2)/SUMF
SUM=SUM+A*F(I2)
SUMSQ=SUMSQ+A*A*F(I2)
33 A=A+1.0
SIGMA=(SUMSQ-SUM*SUM)-1.0/12.0
SIGMA=SQRT(SIGMA)
SMEAN=SUM
Q1=(4.5*F(4)+3.5*F(5)+2.5*F(6)+1.5*F(7)+.5*F(8))/5.0+.5*FIRST
Q2=(4.5 * F(9) + 3.5 * F(10) + 2.5 * F(11) + 1.5 * F(12) + .5 *
1 F(13)) / 5.0 + 5.0*(T(1) +FIRST)
Q3=(4.5 * F(14) + 3.5 * F(15) + 2.5 * F(16) + 1.5 * F(17) +
1 .5 * F(18)) / 5.0 + 5.0*(T(1) + T(2) +FIRST)
PAR1=Q1/Q2
PAR2=Q2/Q3
37 R1=T(1)/T(2)
PRINT 97,C(1),SMEAN
PRINT 99,C(2),SIGMA
PRINT 99,C(3),R1
PRINT 99,C(4),AAA
PRINT 99,C(5),AKK
PRINT 99,C(6),AMM
PRINT 99,CQ(4),PAR1
PRINT 99,CQ(5),PAR2
PRINT 99,CQ(1),Q1
PRINT 99,CQ(2),Q2
PRINT 99,CQ(3),Q3
PRINT 95,CQ(6),F(1)
PRINT 95,CQ(7),F(2)
PRINT 98,CQ(8),F(3)
PRINT 98,CQ(9),FIRST
K=3
DO 102 IN=1,7
N=IN*5+3
M=N-4
PRINT 99,((C(J+K),F(J)),J=M,N)
PRINT 99,C(N+K+1),T(IN)
102 K=K+1
GO TO 999
3 STOP
END
FUNCTION G(X,AKK)
CONS=0.19465/AKK
B=0.1740/AKK
W=0.2881/AKK
G =CONS*EXP(-B*(X-6.06*AKK))-EXP(-W*(X-6.06*AKK))
RETURN
END

```

APPENDIX B: Model Fertility Schedules

The model fertility schedules have been normalized so that total fertility equals 1.0. The rates given in each schedule are age-specific rates per million women in each age interval. These rates are cumulated and divided by 5 for each five-year age interval to provide an average fertility rate for each interval. The tables are arranged by groups of ascending means and subgroups of ascending standard deviation. Within each subgroup of a given mean and standard deviation, R_1 is strictly monotonic. The first ten entries for each schedule are defined as follows:

- 1) $MEAN = \sum_{12.5}^{49.5} a f(a)$
- 2) $STDEV = \sqrt{(\sum_{12.5}^{49.5} a^2 f(a)) - MEAN^2 - 1/12}$
- 3) $R1 = \sum_{15.5}^{19.5} f(a) / \sum_{20.5}^{24.5} f(a)$
- 4) $MED = \hat{a}$ such that $\sum_{12.5}^{\hat{a}} f(a) = 0.5$
- 5) $SKEW = \sum_{12.5}^{49.5} (a - MEAN)^3 f(a) / STDEV^3$
- 6) $PAR\ 1 = \text{average parity (15-19)} / \text{average parity (20-24)}$
- 7) $PAR\ 2 = \text{average parity (20-24)} / \text{average parity (25-29)}$
- 8) $AO = \text{first age of marriage in the nuptiality function}$
- 9) $K = \text{a scale factor, or the time interval after AO during which any given proportion of marriages takes place relative to the standard nuptiality schedule, where } K = 1$
- 10) $M = \text{degree of control of fertility relative to the standard fertility schedule (m in equation (4)).}$

[illegible]

[illegible]

[illegible]

NBA	25-0		25-1		25-2		25-3		25-4		25-5		25-6		25-7		25-8		25-9		25-10		25-11		25-12		25-13		25-14		25-15		25-16		25-17		25-18		25-19		25-20		25-21		25-22		25-23		25-24		25-25		25-26		25-27		25-28		25-29		25-30		25-31		25-32		25-33		25-34		25-35		25-36		25-37		25-38		25-39		25-40		25-41		25-42		25-43		25-44		25-45		25-46		25-47		25-48		25-49		25-50		25-51		25-52		25-53		25-54		25-55		25-56		25-57		25-58		25-59		25-60		25-61		25-62		25-63		25-64		25-65		25-66		25-67		25-68		25-69		25-70		25-71		25-72		25-73		25-74		25-75		25-76		25-77		25-78		25-79		25-80		25-81		25-82		25-83		25-84		25-85		25-86		25-87		25-88		25-89		25-90		25-91		25-92		25-93		25-94		25-95		25-96		25-97		25-98		25-99		25-100		25-101		25-102		25-103		25-104		25-105		25-106		25-107		25-108		25-109		25-110		25-111		25-112		25-113		25-114		25-115		25-116		25-117		25-118		25-119		25-120		25-121		25-122		25-123		25-124		25-125		25-126		25-127		25-128		25-129		25-130		25-131		25-132		25-133		25-134		25-135		25-136		25-137		25-138		25-139		25-140		25-141		25-142		25-143		25-144		25-145		25-146		25-147		25-148		25-149		25-150		25-151		25-152		25-153		25-154		25-155		25-156		25-157		25-158		25-159		25-160		25-161		25-162		25-163		25-164		25-165		25-166		25-167		25-168		25-169		25-170		25-171		25-172		25-173		25-174		25-175		25-176		25-177		25-178		25-179		25-180		25-181		25-182		25-183		25-184		25-185		25-186		25-187		25-188		25-189		25-190		25-191		25-192		25-193		25-194		25-195		25-196		25-197		25-198		25-199		25-200		25-201		25-202		25-203		25-204		25-205		25-206		25-207		25-208		25-209		25-210		25-211		25-212		25-213		25-214		25-215		25-216		25-217		25-218		25-219		25-220		25-221		25-222		25-223		25-224		25-225		25-226		25-227		25-228		25-229		25-230		25-231		25-232		25-233		25-234		25-235		25-236		25-237		25-238		25-239		25-240		25-241		25-242		25-243		25-244		25-245		25-246		25-247		25-248		25-249		25-250		25-251		25-252		25-253		25-254		25-255		25-256		25-257		25-258		25-259		25-260		25-261		25-262		25-263		25-264		25-265		25-266		25-267		25-268		25-269		25-270		25-271		25-272		25-273		25-274		25-275		25-276		25-277		25-278		25-279		25-280		25-281		25-282		25-283		25-284		25-285		25-286		25-287		25-288		25-289		25-290		25-291		25-292		25-293		25-294		25-295		25-296		25-297		25-298		2	
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[illegible]

[illegible]

[illegible]

[illegible]

RED	26.52	26.54	26.54	26.55	26.55	26.56	26.02	26.06	26.10	26.14	26.16	26.16	26.20	26.22	26.25	26.25
SKEN	0.642	0.635	0.628	0.622	0.617	0.612	0.914	0.889	0.864	0.841	0.819	0.798	0.779	0.761	0.745	0.730
PART	0.0412	0.0419	0.0426	0.0434	0.0440	0.0444	0.0160	0.0204	0.0246	0.0286	0.0327	0.0363	0.0398	0.0430	0.0461	0.0493
PAB2	0.2921	0.2923	0.2928	0.2930	0.2938	0.2940	0.3262	0.3281	0.3295	0.3308	0.3326	0.3336	0.3358	0.3379	0.3400	0.3421
MO	15.95	15.94	15.94	15.92	15.93	15.93	17.74	17.43	17.14	16.87	16.61	16.37	16.15	15.96	15.77	15.57
10-14	1.400	1.450	1.500	1.550	1.600	1.650	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700	0.750	0.800
K	3.733	3.785	3.838	3.886	3.935	3.985	1.987	2.031	2.078	2.126	2.175	2.225	2.275	2.325	2.377	2.428
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15-19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	35490	35439	35480	35534	35576	35589	47554	47352	46938	46522	46283	45845	45533	45178	44923	44678
21	52060	51837	51732	51638	51554	51477	72866	70660	68727	67080	65794	64632	63500	62542	61758	61035
22	68775	68435	68218	68002	67818	67640	88036	85613	83494	81639	80089	78649	77413	76288	75331	74448
23	81826	81457	81194	80916	80683	80470	92890	91262	89787	88420	87204	86035	85075	84151	83388	82675
24	88769	88456	88212	87939	87714	87516	90311	89634	88980	88309	87637	87030	86435	85885	85376	84905
25-29	65384	65125	64967	64806	64669	64534	78331	76504	75585	74395	73401	72428	71591	70808	70145	69508
30	92408	91872	91724	91534	91387	91268	84837	84899	84928	84732	84627	84521	84467	84323	84158	84008
31	91751	91755	91746	91687	91663	91664	77971	78498	78986	79376	79637	79928	80129	80323	80468	80628
32	85229	85363	85433	85498	85563	85648	69288	70004	70696	71304	71778	72282	72688	73081	73408	73768
33	77156	77392	77563	77698	77840	77997	61144	61893	62640	63326	63890	64498	65120	65694	66320	66998
34	68506	66112	66292	66452	66619	66775	52670	53326	53995	54627	55156	55726	56217	56702	57120	57620
35-39	82499	82556	82574	82613	82670	82724	69182	69724	70249	70701	71039	71410	71700	71984	72274	72574
40	53268	53268	53405	53537	53659	53784	44254	44751	45270	45770	46188	46608	47006	47442	47779	48124
41	41895	42055	42147	42287	42418	42543	36935	37279	37648	38011	38310	38652	38946	39243	39490	39798
42	32650	32762	32813													

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HEAV STDEV	28-0 5.5	28-0 0.1631	28-0 0.1605	28-0 0.1701	28-0 5.5	28-0 0.1758	28-0 5.5	28-0 0.1808	28-0 5.5	28-0 0.1869	28-0 5.5	28-0 0.1907	28-0 5.5	28-0 0.1921	28-0 5.5	28-0 0.1933	28-0 5.5
1	27.34	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36	27.36
2	0.0682	0.0705	0.0728	0.0751	0.0774	0.0797	0.0820	0.0843	0.0866	0.0889	0.0912	0.0935	0.0958	0.0981	0.1004	0.1027	0.1050
3	0.3327	0.3332	0.3337	0.3342	0.3347	0.3352	0.3357	0.3362	0.3367	0.3372	0.3377	0.3382	0.3387	0.3392	0.3397	0.3402	0.3407
4	14.51	14.39	14.29	14.20	14.11	14.03	13.96	13.89	13.82	13.75	13.68	13.61	13.54	13.47	13.40	13.33	13.26
5	1.100	1.150	1.200	1.250	1.300	1.350	1.400	1.450	1.500	1.550	1.600	1.650	1.700	1.750	1.800	1.850	1.900
6	2.019	2.059	2.099	2.140	2.178	2.216	2.256	2.296	2.336	2.376	2.416	2.456	2.496	2.536	2.576	2.616	2.656
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10-14	35	54	80	105	135	166	197	226	255	285	315	342	366	385	405	425	445
15	544	630	722	802	886	965	1038	1104	1184	1236	1300	1358	1407	1449	1492	1535	1578
16	2221	2387	2561	2702	2848	2980	3100	3204	3342	3412	3511	3599	3672	3733	3797	3861	3925
17	6330	6581	6752	6932	7092	7233	7369	7501	7631	7759	7885	8013	8138	8266	8394	8522	8650
18	13365	13541	13719	13875	14023	14151	14258	14355	14486	14522	14634	14704	14780	14825	14870	14915	14960
19	23705	23732	23817	23876	23955	23882	23869	23855	23870	23926	23954	23977	23971	23953	23950	23950	23950
20-24	9193	9328	9486	9589	9709	9814	9906	9976	10097	10137	10217	10287	10336	10372	10415	10458	10501
25	36233	36063	35979	35820	35714	35614	35517	35395	35388	35257	35200	35154	35081	35006	34945	34884	34823
26	48146	47801	47509	47248	47002	46774	46570	46344	46231	46019	45872	45750	45605	45471	45348	45225	45102
27	59166	58490	58151	57787	57467	57179	56917	56683	56464	56220	56019	55856	55676	55485	55288	55095	54902
28	67196	67405	67563	67758	67906	68054	68204	68354	68431	68592	68680	68779	68886	68905	69006	69096	69186
29	61284	61530	61721	61948	62131	62309	62485	62665	62767	62950	63067	63186	63315	63449	63561	63673	63785
30-34	69539	69615	69658	69733	69771	69816	69871	69923	69925	69996	70005	70036	70070	70121	70146	70166	70186
35	73884	73709	73537	73389	73223	73080	72958	72830	72697	72602	72469	72375	72278	72207	72115	72029	71943
36	74137	74124	74090	74086	74047	74024	74015	74000	73961	73914	73898	73898	73898	73898	73898	73898	73898
37	71196	71307	71379	71484	71547	71616	71693	71766	71832	71896	71940	71988	72053	72089	72125	72161	72197
38	67196	67405	67563	67758	67906	68054	68204	68354	68431	68592	68680	68779	68886	68905	69006	69096	69186
39	61284	61530	61721	61948	62131	62309	62485	62665	62767	62950	63067	63186	63315	63449	63561	63673	63785
40-44	69539	69615	69658	69733	69771	69816	69871	69923	69925	69996	70005	70036	70070	70121	70146	70166	70186
45	54020	54249	54425	54634	54806	54970	55127	55293	55385	55550	55661	55765	55881	55997	56098	56199	56299
46	46938	47134	47280	47458	47607	47745	47875	48018	48129	48230	48327	48410	48508	48601	48686	48771	48856
47	40456	40611	40722	40861	40979	41086	41184	41297	41350	41457	41535	41595	41670	41738	41805	41872	41939
48	34544	34655	34728	34821	34911	34985	35049	35130	35161	35235	35292	35328	35380	35422	35469	35516	35563
49	28684	28765	28815	28887	28951	29004	29049	29091	29131	29186	29232	29277	29327	29372	29417	29462	29507
50-54	41128	41283	41394	41533	41651	41758	41857	41970	42024	42132	42210	42271	42347	42417	42485	42553	42621
55	25294	25346	25371	25417	25458	25490	25514	25557	25563	25599	25632	25648	25671	25688	25715	25742	25769
56	21317	21341	21344	21364	21384	21396	21401	21424	21418	21435	21454	21466	21486	21470	21486	21496	21512
57	17814	17814	17796	17796	17796	17796	17796	17796	17796	17796	17796	17796	17796	17796	17796	17796	17796
58	14708	14689	14657	14639	14625	14605	14580	14551	14528	14507	14480	14457	14438	14419	14400	14381	14362
59	11855	11823	11781	11750	11725	11695	11662	11625	11587	11547	11507	11467	11427	11387	11347	11307	11267
60-64	18198	18203	18190	18193	18198	18196	18187	18196	18181	18184	18193	18183	18186	18180	18186	18186	18186
65	9166	9127	9080	9043	9012	8977	8939	8914	8879	8852	8834	8804	8783	8756	8740	8724	8708
66	6723	6683	6638	6600	6568	6533	6496	6469	6436	6408	6388	6359	6338	6311	6294	6278	6262
67	4741	4704	4664	4630	4600	4568	4535	4501	4481	4455	4437	4411	4391	4367	4351	4335	4319
68	3124	3093	3061	3033	3008	2983	2956	2932	2912	2891	2876	2855	2839	2820	2806	2792	2778
69	1822	1860	1837	1816	1798	1780	1761	1746	1729	1714	1698	1688	1676	1663	1652	1641	1630
70-74	5127	5093	5056	5024	4997	4968	4938	4915	4888	4864	4847	4824	4805	4789	4769	4753	4737
75	1058	1044	1030	1016	1005	993	981	971	961	951	944	935	927	919	912	905	898
76	611	602	593	584	577	570	562	556	550	543	539	533	528	523	519	515	511
77	346	341	335	330	326	322	317	314	310	307	304	301	298	295	293	291	289
78	175	172	169	167	164	162	160	158	156	154	153	151	150	149	147	146	145
79	47	46	45	44	44	44	44	44	44	44	44	44	44	44	44	44	44
80-84	447	441	434	428	423	418	413	408	404	399	396	392	389	385	382	380	377

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RED	30.75	30.75	30.76	30.84	30.85	31.57	31.59	31.61	31.62	31.64	31.65	31.67	31.68	31.65
SKEN	0.122	0.118	0.113	0.109	0.085	0.078	0.252	0.229	0.219	0.209	0.201	0.191	0.183	0.176
PART	0.1014	0.1038	0.1058	0.1230	0.1280	0.0074	0.0098	0.0123	0.0149	0.0175	0.0201	0.0228	0.0252	0.0276
PAR2	0.3254	0.3254	0.3258	0.3262	0.3854	0.3862	0.1807	0.0859	0.1899	0.1940	0.1975	0.2009	0.2042	0.2095
AO	12.92	12.81	12.70	12.96	12.69	12.67	17.70	17.48	17.25	17.05	16.86	16.52	16.37	16.30
K	1.350	1.400	1.450	1.500	0.800	0.850	0.700	0.650	0.700	0.750	0.800	0.850	0.900	1.000
H	0.646	0.674	0.701	0.728	0.011	0.033	0.305	0.338	0.372	0.407	0.442	0.477	0.513	0.549
12	0	0	1	3	6	0	0	0	0	0	0	0	0	0
13	70	88	105	124	65	110	0	0	0	0	0	0	0	0
14	374	417	454	497	474	601	0	0	0	0	0	0	0	0
10-14	89	101	112	125	108	143	0	0	0	0	0	0	0	0
15	1149	1224	1284	1356	1730	1956	0	0	0	0	1	1	2	3
16	2762	2864	2940	3036	4438	4705	2	4	9	17	28	42	60	80
17	5953	5700	5769	5871	8894	9092	39	78	127	190	261	342	430	517
18	9892	9711	10003	10085	14870	14904	383	579	782	1001	1214	1431	1647	1842
19	15186	15208	15779	15215	21042	20889	2172	2680	3114	3530	3891	4231	4548	4809
15-19	6916	6993	7035	7113	10195	10369	519	668	806	948	1079	1209	1338	1450
20	21121	21076	20984	20965	26736	26448	7378	7937	8445	8852	9166	9458	9723	9915
21	27169	27058	26906	26832	33778	33521	15676	15996	16150	16302	16381	16468	16552	16574
22	33103	32951	32765	32652	35962	35695	25534	25183	24989	24766	24595	24452	24272	24126
23	38609	38440	38245	38113	39421	39112	33506	34761	34199	33730	33270	32560	32218	31927
24	43089	43225	43043	42910	42194	41957	34556	42843	42153	41557	40988	40504	40074	39648
20-24	32678	32550	32389	32294	35218	34909	25502	25406	25226	25086	24914	24783	24672	24406
25	47539	47408	47266	47156	49358	49211	49899	48674	48106	47563	47093	46666	46250	45887
26	50990	50812	50729	50661	45957	45845	54521	54019	53548	53116	52701	52336	51998	51671
27	53065	53038	53012	52986	46908	46908	56914	56649	56392	56181	55918	55703	55497	55316
28	54523	54551	54587	54607	47490	47589	57623	58558	58502	58321	58158	57939	57723	57509
29	54975	55042	55121	55172	47792	47938	58935	59027	59136	59214	59286	59349	59442	59491
25-29	52198	52170	52143	52117	46483	46498	55856	55567	55302	55052	54810	54597	54396	54203
30	54373	54456	54556	54619	47732	47900	58226	58424	58638	58813	58980	59125	59247	59373
31	53040	53132	53243	53311	47243	47420	58735	57012	57289	57524	57752	57950	58123	58301
32	51338	51428	51542	51609	46591	46764	54953	55235	55537	55797	56054	56278	56475	56679
33	49229	49311	49418	49476	45694	45852	52808	53081	53376	53633	53890	54118	54309	54515
34	46854	46933	47039	47096	44344	44488	50296	50550	50831	51077	51329	51546	51741	51949
30-34	50967	51052	51159	51222	46321	46485	54607	54860	55134	55369	55601	55802	55979	56163
35	44211	44281	44378	44428	42791	42913	47590	47808	48054	48270	48497	48691	48867	49058
36	41248	41302	41385	41422	40934	41028	44614	44781	44977	45151	45338	45495	45638	45798
37	38147	38182	38246	38267	38904	38968	41538	41648	41768	41910	42049	42161	42263	42384
38	34845	34955	34902	34905	33651	33647	34463	34556	34699	34852	35014	35178	35347	35511
35-39	37902	37936	37997	38016	38575	38635	41295	41402	41539	41657	41792	41899	41977	42113
40	26551	26526	26525	26495	29720	29687	29751	29721	29686	29672	29640	29599	29574	29544
41	21524	21486	21468	21427	24927	24873	24802	24725	24624	24524	24420	24306	24182	24059
42	16771	16725	16697	16652	20121	20054	19352	19244	19154	19064	18982	18899	18831	18768
43	12204	12159	12127	12083	15184	15116	14302	14194	14100	14007	13928	13842	13760	13671
44	8118	8080	8050	8012	10085	10025	9672	9579	9496	9414	9343	9267	9196	9133
40-44	17034	16995	16973	16934	20088	20031	19523	19432	19359	19285	19227	19160	19095	19046
45	4971	4943	4921	4894	6628	6583	6006	5938	5876	5815	5761	5705	5652	5605
46	3082	3062	3046	3028	4218	4185	3765	3717	3673	3629	3591	3551	3513	3480
47	1823	1811	1802	1791	2531	2510	2239	2208	2180	2153	2128	2103	2080	2058
48	947	941	936	930	1326	1315	1166	1149	1134	1119	1106	1092	1080	1068
49	258	256	255	253	362	359	317	312	308	304	297	293	290	287
45-49	2216	2216	2216	2216	2216	2216	2216	2216	2216	2216	2216	2216	2216	2216

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POPULATION COUNCIL
DEMOGRAPHIC DIVISION GRANTS

The Population Council has published a new description of its Demographic Division grants program, including detailed information concerning the topics on which applicants are encouraged to focus in 1974 and 1975.

Under the Demographic Division grants program support is available for research, institutional development, and fellowships. In each of these categories, the subject matter may fall in one or more of the following fields of interest:

- Demographic processes and structure—in particular, levels and trends of population growth, fertility, mortality, migration, and the composition and spatial distribution of populations.
- The antecedents of demographic processes—in particular, the economic, social, and psychological determinants of demographic behavior.
- The effects of population processes—in particular, their economic, social, and environmental consequences, and their nature, incidence, and timing.
- Population policy—in particular, social and political responses to the effects of population processes and the analysis of possible policy choices that seek